

A Simple Mathematical Solution to the Cosmological Constant Problem

Stéphane Wojnow 

Independent researcher, 7 avenue Georges Dumas, 87 000 Limoges, France

ABSTRACT

Assuming an energy density of the cosmological constant in Quantum Field Theory, we propose a simple mathematical solution to the cosmological constant problem, i.e., the disagreement of the order of a factor 10^{122} between the theoretical and the measured value of the vacuum energy density. We try to give a non-exclusive route for our solution to make physical sense.

Keywords: Cosmological constant problem-Vacuum catastrophe- Cosmological constant- zero point energy- Hildebrand solubility parameter.

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INTRODUCTION

In cosmology, the cosmological constant problem or vacuum catastrophe is the disagreement between the observed values of vacuum energy density (as a small value) and the theoretical value (as a large value) of zero-point energy suggested by Quantum Field Theory (QFT).

Depending on the Planck energy cutoff and other factors, the discrepancy is as high as 120 orders of magnitude [1] as a worst theoretical prediction in the history of physics [2]. The purpose of this paper is to postulate a constant cosmic energy density in QFT.

For this we proposed a simple mathematical solution to the cosmological constant problem, i.e. the 10^{122} order mismatch between the theoretical value and the vacuum energy density

measurement. We try to provide a non-exclusive path to our solution so that it makes physical sense.

MATHEMATICAL SOLUTION

Here we define parameters with m_p as Planck mass, l_p Planck length, \hbar reduced Planck constant, c as the speed of light in vacuum, Λ as cosmological constant, A energy density of the zero point energy in quantum field theory [3], B as energy density of the vacuum assumed for the cosmological constant in QFT. Moreover C is the cosmological constant's energy density of the Λ CDM model, and finally, A/C is the usual value of the vacuum catastrophe ("cosmological constant problem"), equal about 10^{122} .

We will show that $A/C = C/B$ so that $C^2 = AB$. Let us now to consider A as the energy density of the zero-point expressed in J/m^3 and then have,

$$A = \frac{m_p c^2}{l_p^3} \quad (1)$$

and in term of reduced Planck constant is

$$A = \hbar(l_p^{-4})c \quad (2)$$

or

$$A = \hbar(l_p^{-2})^2 c \quad (3)$$

and for B , the energy density of the vacuum assumed for the cosmological constant in QFT we have,

$$B = \frac{1}{(8\pi)^2} \hbar(\Lambda_{m^{-2}})^2 c \quad (4)$$

with

$$m_p l_p = \frac{\hbar}{c} \quad (5)$$

so, the energy density of the cosmological constant C , with J/m^3 , is the geometric mean of A and B ,

$$\frac{A}{C} = \frac{C}{B} \quad (6)$$

$$C = \sqrt{AB} = \sqrt{\hbar(l_p^{-2})^2 c \hbar(\Lambda_{m^{-2}})^2 \frac{c}{(8\pi)^2}} \quad (7)$$

$$C = \sqrt{AB} = \sqrt{\hbar^2 (l_p^{-2})^2 \frac{c^2 (\Lambda_{m^{-2}})^2}{(8\pi)^2}} \quad (8)$$

and

$$C = \frac{\hbar c (\Lambda_{m^{-2}})}{l_p^2 (8\pi)} \quad (9)$$

$$C = \frac{F_p (\Lambda_{m^{-2}})}{8\pi} \quad (10)$$

where here we define the Planck force as,

$$F_p = \frac{c^4}{G} \quad (11)$$

and then C is

$$C = \frac{c^4 (\Lambda_{m^{-2}})}{8\pi G} = \rho \Lambda c^2 \quad (12)$$

In the other words, the classical formula of the energy density of the cosmological constant in the Λ CDM (Lambda-CDM) model. With this addition to simplify the verification

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \quad (13)$$

or

$$l_p^2 = \frac{\hbar G}{c^3} \quad (14)$$

so

$$\frac{\hbar c}{l_p^2} = \frac{\hbar c^4}{\hbar G} = \frac{c^4}{G} = F_p \quad (15)$$

CONCLUSION

The method used here is as follows. We write density energy in J/m^3 of the zero energy in the QFT, A with the reduced Planck constant to make appear a unit of dimension [L^{-2}]. Here we assume an energy density of the cosmological constant in QFT, B always with \hbar as reduced Planck constant, on the same dimensional model as A . (The cosmological constant of dimension [L^{-2}] is of the same dimension as lp^{-2}).

It is shown that the energy density of the cosmological constant of general relativity, C , is the geometric mean of A and B . This could validate the hypothetical energy density of the cosmological constant in QFT. However, a question arises here is that: What is the physical meaning of the square root of an energy density (for A or B)?

There is no reference to this for cosmology. But there is one for the cohesive energy density (CED) in connection with perfect gases. In fact, the cohesive energy density is the energy needed to completely remove the unit volume of molecules from their neighbors to infinite separation (an ideal gas).

This is equal to the heat of vaporization of the compound divided by its molar volume in the condensed phase. In order for a material to dissolve, these same interactions need to be overcome, as the molecules are separated from each other and surrounded by the solvent.

In 1936 Joel Henry Hildebrand suggested the square root of the cohesive energy density as a numerical value indicating solvency behavior[4].

This later became known as the Hildebrand solubility parameter. Materials with similar solubility parameters will be able to interact with each other, resulting in solvation, miscibility, or swelling.

Of course, to state of vacuum solubility of the QFT in the quantum vacuum of the cosmological constant is a physical nonsense. On the other hand, by noting that the value of the energy density of the QFT vacuum is exactly that of the energy density of the Planck mass as;

$$\frac{m_{Pl}c^2}{l_{Pl}^3} \quad (16)$$

The solubility of the latter in the quantum vacuum of the cosmological constant makes physical sense and with Hildebrand solubility parameter ultimately means an ideal gas. More precisely, with my proposal, the constant cosmic energy density, i.e. its pressure, was achieved.

REFERENCES

- [1] R. J. Adler, B. Casey, and O. C. J. A. J. o. P. Jacob, "Vacuum catastrophe: An elementary exposition of the cosmological constant problem," vol. 63, no. 7, pp. 620-626, 1995.
- [2] M. P. Hobson, G. P. Efstathiou, and A. N. Lasenby, *General relativity: an introduction for physicists*. Cambridge University Press, 2006.
- [3] L. J. P. L. B. Lombriser, "On the cosmological constant problem," vol. 797, p. 134804, 2019.
- [4] J. J. S. P. T. Burke, Application, T. Book, and D. Paper Group of the American Institute for Conservation: Washington, USA, "Part 2. Hildebrand Solubility Parameter," 1984.